Microwave Filter Design Using an Electronic Digital Computer*

LEO YOUNG†

Summary—It is shown how a transmission-line circuit can be analyzed by a digital computer. Transformation matrices are used and broken down into equations which are applicable to a computer.

"Synthesis by computer" involves feeding in an approximate design and programming the computer to search for better parameters until the performance matches the specification. Examples are given to indicate time and cost of both analysis and synthesis procedures on an IBM Type 650 digital computer.

The synthesis of a stagger-tuned three-cavity filter is described.

INTRODUCTION

APPROXIMATE design formulas are available for various microwave circuits, such as direct-coupled resonator filters,[1] where the performance in the pass band is specified. To investigate the performance outside the pass band (such as the rejection at higher and lower frequencies, and the location of higher order pass bands), to determine the effect of tolerances, and to test the accuracy of the design formulas, several microwave circuits have been analyzed over wide frequency bands by programming a digital computer. The circuits analyzed in this paper are all direct-coupled multicavity resonator filters exhibiting band-pass behavior.

The synthesis of distributed parameter circuits is more difficult than their analysis. There appears to be one exact method,[2] but it holds only for resistor-transmission-line circuits. Another method of synthesis is described in this paper. Its main usefulness is likely to be in those cases where it is desired to improve an already existing approximate design. This method does not develop algebraic formulas, but arithmetical procedures which are programmed for a digital computer. The design procedure may be broken down into the following steps.

1) The designer estimates his circuit parameters by approximate design formulas, or by means of a Smith chart.
2) He analyzes the resulting rough design on a computer.
3) He varies his parameters until the new performance is acceptable.

Step 3 is usually accomplished by a series of approximations, in which the design adjustments become smaller and smaller as the performance converges on the specification.

The computer is able to reach decisions that make "synthesis by computer" possible. Essentially, this "synthesis" consists in letting the computer not only perform the arithmetical tasks in step 2 above, but also take over the decision making listed in step 3. It is in this narrow sense only that "synthesis by computer" is spoken of in this paper.

Three questions now present themselves.

1) Is the specification realizable with the given circuit configuration?
2) Can the prescribed process of cut and try (adjustment and selection) converge to give the specified performance?
3) Can initial approximate parameter values be found so that the process not only can, but *does* converge?

Only if all three answers are positive, will the search for a better design be successful. No universal test can be applied, and it may be a matter of trial and error until a convergent combination is found.

TIME AND COST CONSIDERATIONS

When should a digital computer be used? Briefly, whenever lengthy repetitive calculations are involved, as in the computation of the VSWR of a filter at many frequencies. To gain some idea of the time and cost involved, the following example is given. The machine was an IBM Type 650, a medium fast computer. A basic language program was used, and the computer had a floating decimal device and three index registers. It also had a 60-word magnetic core storage, and bands of data and instructions were transferred from the drum to core storage for execution.

It was required to design a three-cavity direct-coupled transmission line band-pass filter. An optimum (equal ripple) design was specified. To program the computer for the analysis of this circuit took about two days. To run one analysis on the machine takes just three seconds per frequency point, plus one minute loading time. Thus, 40 spot frequencies take three minutes. To synthesize the circuit by trial and error, for example, might involve five adjustments by the operator. Each time, the computed data has to be examined before the next trial solution is loaded into the computer. This particular problem was then programmed for computer synthesis. The computer completed the synthesis in five to six minutes, including the final analy-

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† Westinghouse Electric Corp., Baltimore 3, Md.

[1] S. B. Cohn, "Direct-coupled resonator filters." PROC. IRE, vol. 45, pp. 187–196; February, 1957.

[2] P. I. Richards, "Resistor-transmission-line circuits," PROC. IRE, vol. 36, pp. 217–220; February, 1948.

sis. Computer time is a dollar a minute in round figures, and there was actually a net saving over the repeated analysis method. However, to program the computer for synthesis took about two weeks, as against two days for the analysis alone.

TRANSFORMATION MATRICES

When using a computer, advantage has to be taken of its great facility to perform repetitive calculations. It is best, therefore, to break the problem down into a number of operations of similar form, letting the machine substitute new numbers as it works its way through the problem. This can be understood by visualizing a transmission line containing lumped discontinuities (such as irises or sudden changes in cross section). The transmission line in Fig. 1 is loaded at intervals by obstacles (represented by rectangular boxes). For reasons which are explained presently, we shall always work from output (on the right) to input (on the left). The several vertical broken lines in the figure are the reference planes in the waveguide. The "state" of the waveguide in the various reference planes is denoted by S_1, S_2, S_3, $\cdots$. "State" means any two complex numbers forming a two-dimensional complex column vector, which completely describes the electromagnetic condition or state of the waveguide, assuming only one mode of propagation. The two elements of the vector representing a state are usually the voltage and current,[3] respectively, or the incident and reflected wave amplitudes.[3,4] The state S_2 in reference plane 2 can be calculated from the state S_1 in reference plane 1 by operating on S_1 with the matrix operator T_1:

$$S_2 = T_1 S_1. \tag{1}$$

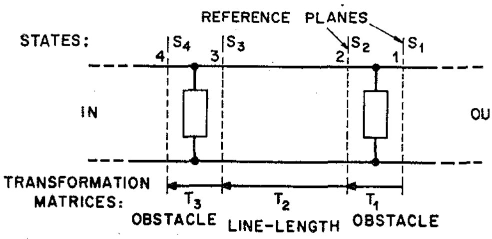

Fig. 1—Transmission line loaded at intervals.

If, for instance, S is composed of voltage E and current I, so that

$$S = \begin{pmatrix} E \\ I \end{pmatrix} \tag{2}$$

then T is the $ABCD$ matrix[3]

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{3}$$

The computer then solves for S_{n+1} by an iterative procedure from

$$S_{n+1} = T_n T_{n-1} \cdots T_3 T_2 T_1 S_1. \tag{4}$$

It is more convenient to work back from the output to the input because the state S_1 is always known. Usually, the termination is a matched load, and (after normalization)

$$S_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{5}$$

for the $ABCD$ matrix.

SYNTHESIS BY COMPUTER—ADVANTAGES AND LIMITATIONS

At first, to save time and constant reloading, several trial circuits were loaded together, and the machine analyzed one after another. From the computed results a new and, if possible, better set of circuit parameters was estimated, and so on. In many cases, it was found that data accumulated at a greater rate than the human operator's limited capacity to absorb and select. It was then that the idea of letting the computer not only calculate but also select from the data, finally printing only the best design, seemed most attractive.

Certain limitations of the search or synthesis procedure by computer later became apparent. To write the specification exactly as desired into the computer is not always practical. For instance, to specify a certain maximum VSWR inside a given frequency band would require more time programming and computing than to specify the n frequencies at which the reflection coefficient of a lossless n-cavity filter is to be zero, plus one other parameter to fix the scale. For instance, one can let the n frequencies of zero reflection coefficient be determined by a comparison with the corresponding Tchebycheff polynomial having the desired bandwidth. Thus the formulation of the problem will usually involve some compromise; the specification should be written so that it can be programmed readily into the computer, while yet retaining the intentions of the designer.

SYNTHESIS OF A THREE-CAVITY STAGGER-TUNED FILTER

The circuit to be optimized is a symmetrical lossless three-cavity direct-coupled band-pass filter, as shown in Fig. 2, consisting of a transmission line with four shunt inductive posts. Thus, there are four independent parameters, the values at some fixed frequency of the susceptances $-jb_1$, $-jb_2$ and their spacings ϕ_1, ϕ_2. Since it would be difficult to adjust and optimize four independent parameters, they will be reduced to two. To begin

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[3] G. L. Ragan, "Microwave Transmission Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 9, pp. 544–554; 1948.

[4] L. Young, "Branch guide directional couplers," *Proc. Natl. Electronics Conf.*, vol. 12, pp. 723–732; 1956. (See also *Proc. IEE*, pt. B, vol. 104, p. 586; November, 1957.)

with, they are reduced to three by keeping b_1 fixed. (If necessary, more values of b_1 can be tried later.)

The synthesis of the circuit now is explained briefly with the aid of Fig. 2(b), in which

$$\phi_1 = \tfrac{1}{2}(\beta_1 + \theta_1) \tag{6}$$

where at the plane separating β_1 and θ_1 the admittance is taken real at all frequencies. This defines β_1; θ_1 is made variable and later optimized by the computer.

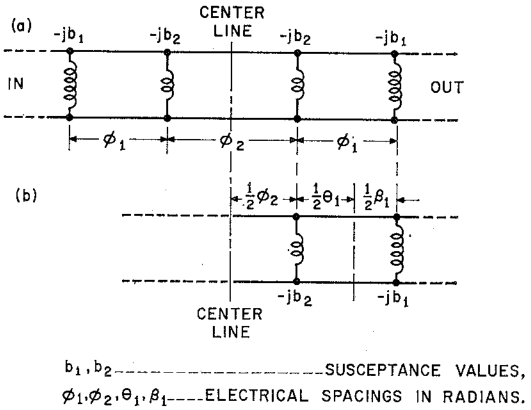

Fig. 2—Symmetrical three-cavity filter.

The susceptance value b_2 is the other variable to be optimized by the computer. If the filter is to be matched at this frequency, then the susceptance value seen from the center plane must be zero, which then determines ϕ_2.

So far, only one frequency has been considered, and θ_1 and b_2 are still arbitrary. Let it be required that the filter also be matched at two other frequencies, one on each side of the fixed frequency. It is required that b_2 and θ_1 be adjusted until a perfect match is obtained at the three given frequencies.

The design procedure, or synthesis by computer, is now as follows. A value of θ_1 and b_2 is estimated from an approximate design formula.[1,5–7] With this as a starting point, the computer keeps θ_1 fixed and solves for b_2 twice, once for $b_2 = b_2'$ which gives a perfect match at the lower specified frequency, and once for $b_2 = b_2''$ which gives a perfect match at the upper specified frequency. This is repeated for several values of θ_1. The design is complete when

$$b_2' - b_2'' = D(\theta_1) = 0. \tag{7}$$

Any frequency dependence can be assumed for the shunt susceptances. The problem of interest concerned a coaxial line with posts between the inner and outer conductors which behave as shunt inductances. Their reactance therefore is made proportional to frequency.

It is important to make the first estimate as accurate as possible; otherwise, the procedure may not converge. This occurs when the initial value is on the wrong side of a local maximum or minimum of the quantity which it is tried to zero (point A in Fig. 3). The increment values should be chosen carefully. In this problem, it was considered better to use two sets of increments. First, the steps are larger until the appropriate zero line is crossed (points 1, 2, 3, 4 in Fig. 3). Then this is repeated in smaller steps, starting from the hitherto best value (points 4, 5, 6, 7, 8 in Fig. 3). Finally, the machine interpolates between the two nearest points (7 and 8 in the figure) on opposite sides of the zero line.

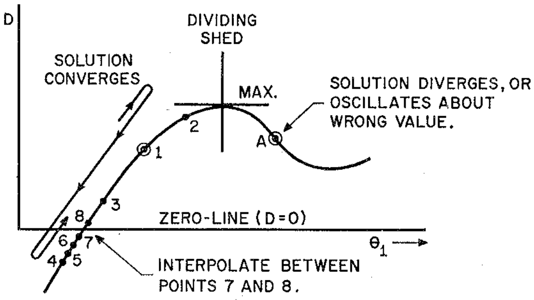

Fig. 3—Illustrating "synthesis by computer."

CONCLUSION

Most of the numerical and experimental work has been undertaken in connection with the design of three and four-cavity coaxial-line filters. The cavities are defined by posts between the inner and outer conductors, which behave as shunt inductances. The programs also could be applied to dispersive lines in which the shunt susceptances are proportional to the guide wavelength (this holds quite closely for waveguide inductive irises). In this case, frequency is replaced by reciprocal guide wavelength. Programs are now available which analyze direct-coupled resonator filters with up to nine cavities, computing the VSWR, attenuation, and other quantities of interest at all specified frequencies. Small ohmic losses, both distributed in the line and lumped with the inductances, also may be included with these programs.

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[5] G. C. Southworth, "Principles and Applications of Waveguide Transmission," D. Van Nostrand Co., Inc., New York, N. Y., ch. 9; 1950.

[6] R. Levy, "An improved design procedure for the multi-section generalized microwave filter," *Proc. IEE*, pt. C, vol. 104, pp. 423–432; September, 1957.

[7] H. Seidel, "Synthesis of a class of microwave filters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 107–114; April, 1957.